

Robust Strategies and Counter-Strategies

Building a Champion Level Computer Poker Player

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University of Alberta
Computer Poker Research Group

One Sentence Summary

How can we create a poker program for competing against expert players?

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How can we create a poker program for competing against expert players?

- Three new techniques for finding game theoretic strategies
- Useful for poker, applicable to other domains
- Show the value of these approaches through competitions against expert humans and computers

- 1 Introduction
- 2 Playing to Not Lose: Counterfactual Regret Minimization
- 3 Playing to Win: Frequentist Best Response
- 4 Playing to Win, Carefully: Restricted Nash Response
- 5 Competition Results
- 6 Conclusion

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The Computer Poker Research Group



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- Martin Zinkevich and I collaborated on this work

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- Martin Zinkevich and I collaborated on this work
 - This is a huge understatement

Texas Hold'em Poker



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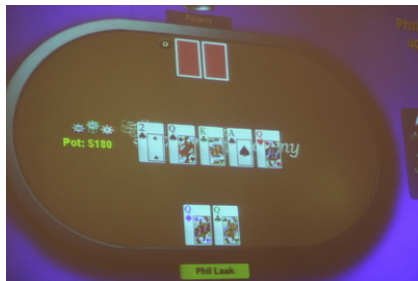
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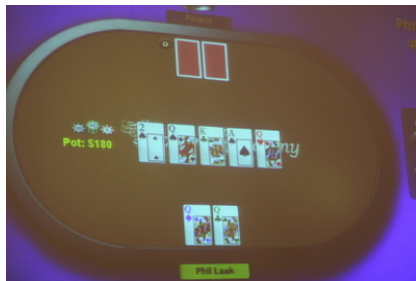
- Poker is a collection of wagering card games
- Texas Hold'em is considered to be the most strategic variant
- Players play a series of short games against each other
- Goal: Win as much money as possible from opponents over this series of games

Heads-Up Texas Hold'em Poker



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- As the game progresses, more cards are revealed
 - Private cards that only one player can see and use
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- Players alternate taking actions:
 - Bet: Make a wager that their cards will be the best
 - Call: Match the opponent's wager
 - Fold: Surrender this game, and begin a new one.

Heads-Up Texas Hold'em Poker



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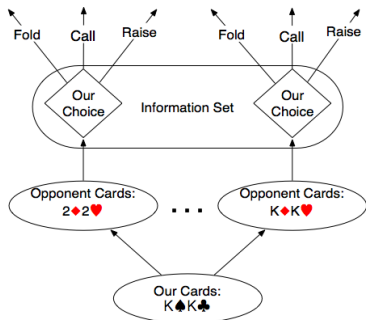
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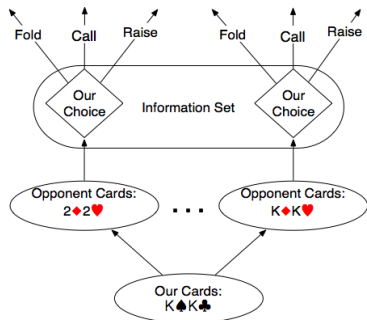
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- Our techniques are applicable beyond poker

Strategies and Information Sets



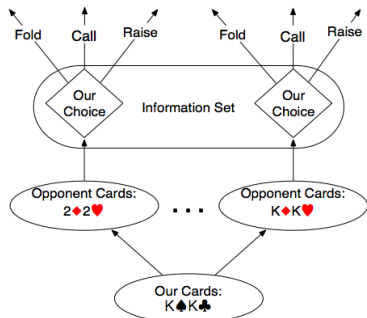
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Strategies and Information Sets



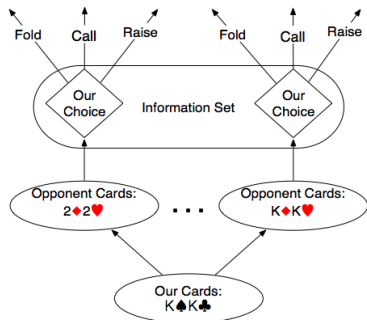
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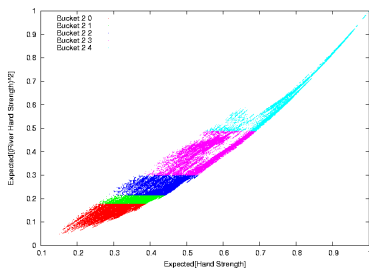
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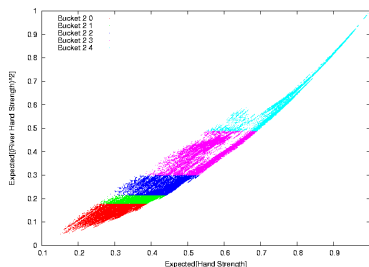
- Because of hidden information, some game states are indistinguishable
- An *information set* is a set of game states that we cannot tell apart
- We have to play the same way for every game state in an information set
- A *behavioral strategy* is a probability distribution over actions for each information set

Computer Poker



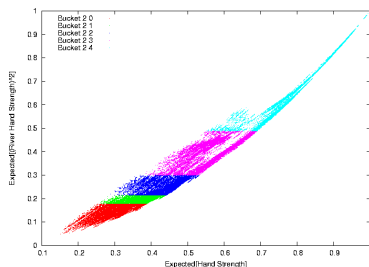
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Computer Poker



- Poker is big — 10^{18} game states
- We abstract the cards into buckets to make the size more reasonable — 10^{12}
- Poker strategies for the abstract game are still powerful in the “real” game, but there is a loss

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Counterfactual Regret Minimization

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- Nash Equilibrium: strategy for each player, where no player can do better by unilaterally changing their strategy
- Approximation to a Nash equilibrium: no player can do better than ϵ by switching

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- Counterfactual Regret Minimization requires memory proportional to number of *information sets* — much smaller.
- Poker has $3.16 * 10^{17}$ game states and $3.19 * 10^{14}$ information sets

Counterfactual Regret Minimization: Theory

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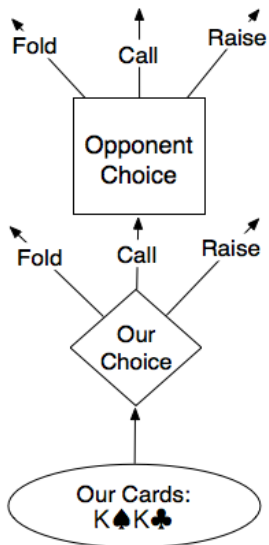
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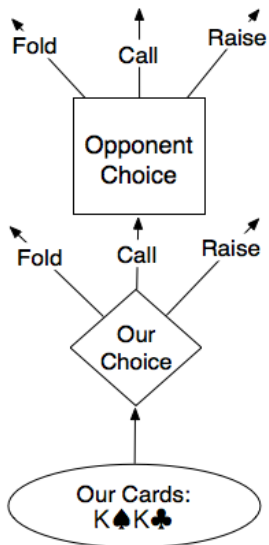
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- How do we minimize Average Overall Regret?

Immediate Counterfactual Regret



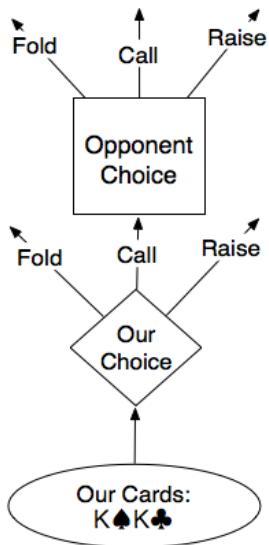
- Break down overall regret into the regret for each action at each information set

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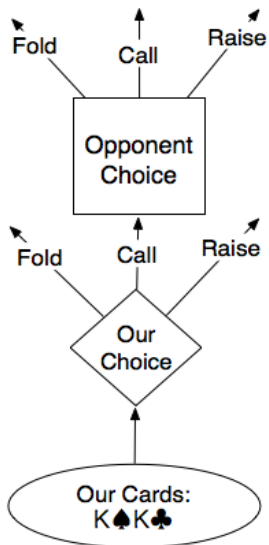
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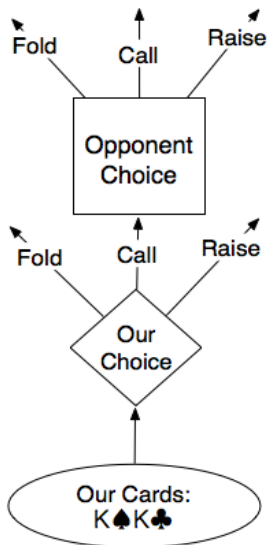
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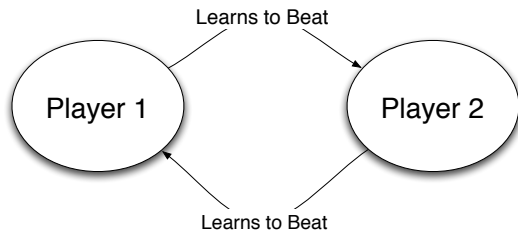
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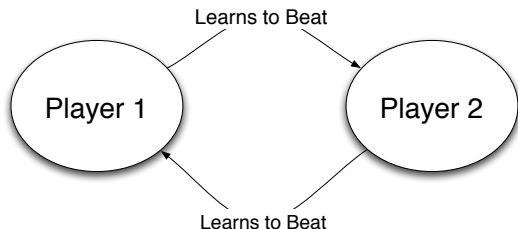
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- Immediate Counterfactual Regret: Weight this regret by the probability of the opponent reaching the information set
- Average Overall Regret is less than the sum of Immediate Counterfactual Regret
- So, if we can minimize our immediate counterfactual regret *at each information set*, then we approach a Nash equilibrium

Counterfactual Regret Minimization: Basic Idea



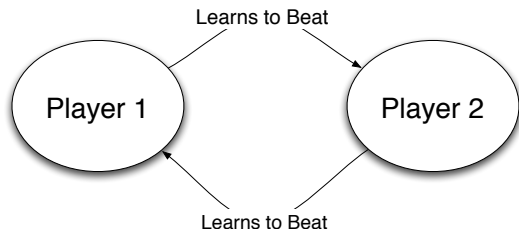
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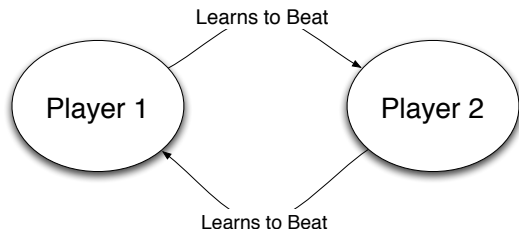
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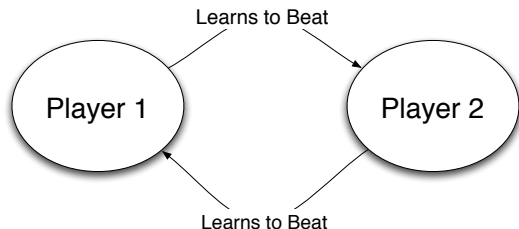
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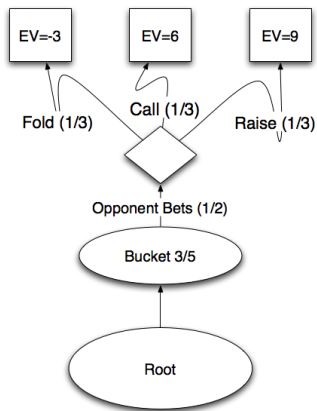
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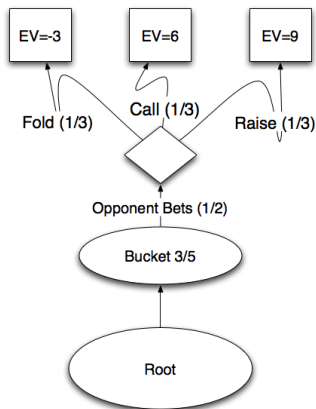
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- How do we update the action probabilities after each game?

Counterfactual Regret



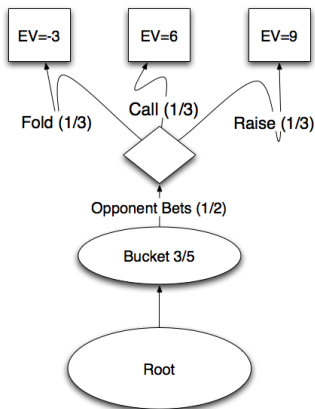
Counterfactual Regret

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- Strategy's EV: 4

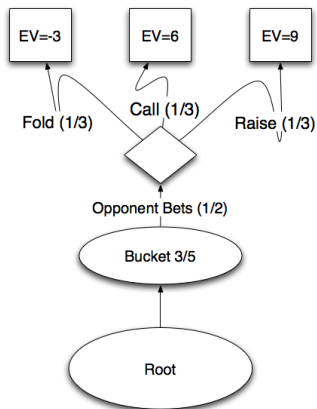
Counterfactual Regret



- Compute expected value of each action
- Calculate the *regret* for not taking each action
- (Regret: Difference between the EV for taking an action and the strategy's EV)

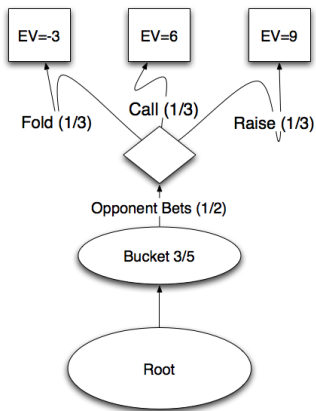
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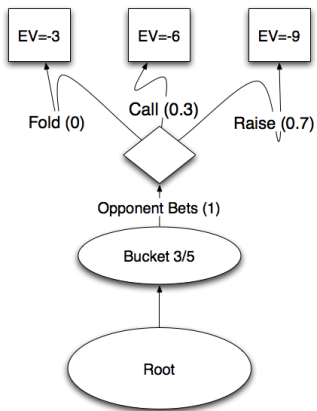
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 - Add up Counterfactual Regret over all games
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- Strategy's EV: 4
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 - Total CFR: (-3.5, 1, 2.5)

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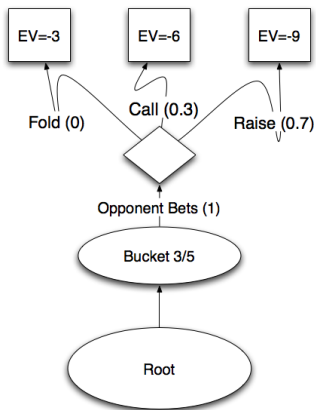


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- Add up Counterfactual Regret over all games
- Assign new probabilities proportional to accumulated positive CFR
- Strategy's EV: 4
- Regret: (-7, 2, 5)
- Total CFR: (-3.5, 1, 2.5)
- New Probabilities: (0, 0.3, 0.7)

Counterfactual Regret Example 2

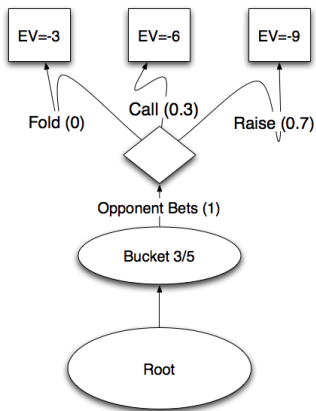


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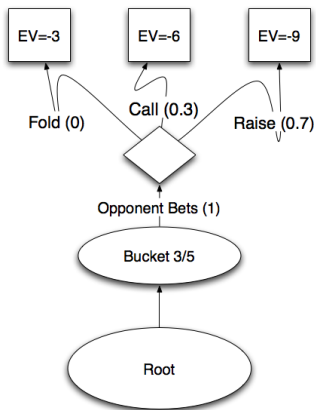
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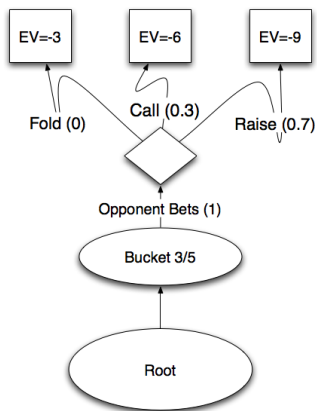
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- Strategy's EV: -8.1
- Regret: (5.1, 2.1, -0.9)
- Total CFR: (1.6, 3.1, 1.6)
- New Probabilities: (0.25, 0.5, 0.25)

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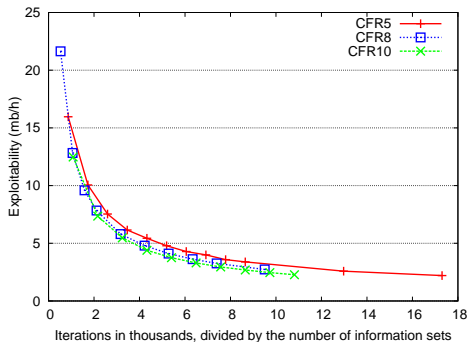
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 - Poker: # iterations grows *linearly* with # information sets
 - (Because seeing a few samples of the states in an information set is enough to choose a good strategy for that information set)
- In practical terms: we can solve very large games (10^{12} states) in under two weeks
- That's two orders of magnitude larger than was previously possible

Convergence to a Nash Equilibrium



| Abstraction | Size (game states) ($\times 10^9$) | Iterations ($\times 10^6$) | Time (h) | Exp (mb/h) |
|-------------|--------------------------------------|------------------------------|----------|------------|
| 5 | 6.45 | 100 | 33 | 3.4 |
| 6 | 27.7 | 200 | 75 | 3.1 |
| 8 | 276 | 750 | 261 | 2.7 |
| 10 | 1646 | 2000 | 326 | 2.2 |

Comparison to the 2006 AAI Competition

| | Hyperborean | Bluffbot | Monash | Teddy | Average |
|--------------|-------------|----------|--------|-------|---------|
| Smallbot2298 | 61 | 113 | 695 | 474 | 336 |
| CFR8 | 106 | 170 | 746 | 517 | 385 |

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- "Playing to Not Lose"

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Frequentist Best Response

- Best Response: best possible counter-strategy to some strategy

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- Best Response: best possible counter-strategy to some strategy
- Useful for a few reasons:
 - Tells you how exploitable that strategy is
 - Could use it during a match to win

Best Response Challenges

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- abstract game best response is easy, but has some challenges:
 - Need to actually have the opponent’s strategy
 - Resulting counter-strategy plays in the same abstraction as the strategy

Best Response Challenges

- “real” best response is intractable
- abstract game best response is easy, but has some challenges:
 - Need to actually have the opponent’s strategy
 - Resulting counter-strategy plays in the same abstraction as the strategy
- (Bigger abstraction == better counter-strategy)

Motivating Frequentist Best Response

- We'd like to make best response counter-strategies with fewer restrictions:
 - What if we don't have the actual strategy, only observations?
 - What if we want to choose the abstraction that the counter-strategy uses?

Frequentist Best Response: Basic Idea

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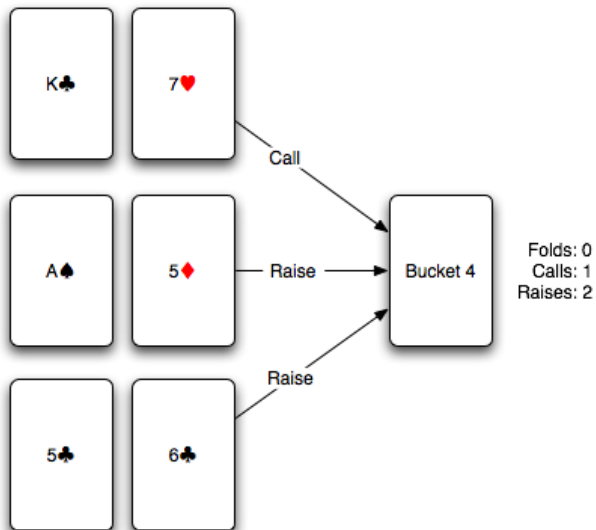
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- Construct an opponent model, where action probabilities are just the action frequencies
- Find the abstract game best response to the opponent model
- Use the counter-strategy to play against the strategy in the real game

Abstracting the data



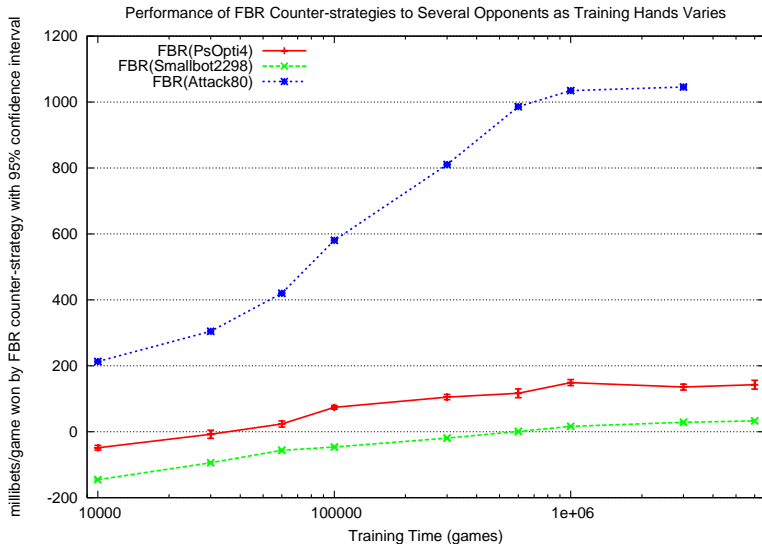
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Frequentist Best Response

- There's a few variables you need to get right:
 - Who is the strategy playing against for the million hands? (Self play is bad, because it doesn't explore the whole strategy space)
 - What do you do in states you never observe? (We assume they call)

Frequentist Best Response



Frequentist Best Response

| | PsOpti4 | PsOpti6 | Attack60 | Attack80 | Smallbot1239 | Smallbot1399 | Smallbot2298 | CFR5 | Average |
|------------------|---------|---------|----------|----------|--------------|--------------|--------------|------|---------|
| FBR-PsOpti4 | 137 | -163 | -227 | -231 | -106 | -85 | -144 | -210 | -129 |
| FBR-PsOpti6 | -79 | 330 | -68 | -89 | -36 | -23 | -48 | -97 | -14 |
| FBR-Attack60 | -442 | -499 | 2170 | -701 | -359 | -305 | -377 | -620 | -142 |
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Frequentist Best Response

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- Columns are poker strategies we've produced in the past
- Rows are counter-strategies to each strategy
- CFR5 is a Counterfactual Regret Minimization strategy
- Two observations:
 - The diagonal has the matches where the counter-strategy plays against its intended opponent. These scores are all good - significantly higher than the CFR strategy does
 - Everything off the diagonal is horrible

Frequentist Best Response: Conclusions

- "Playing to Win"

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Frequentist Best Response: Conclusions

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- We also use them to evaluate our strategies, to see how weak they are
- However, they are *brittle* — when used against other opponents, even weak ones, they can lose badly.
- Is there a way to keep the exploitiveness of FBR counter-strategies, while also gaining the robustness of CFR strategies?

- 1 Introduction
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- 3 Playing to Win: Frequentist Best Response
- 4 Playing to Win, Carefully: Restricted Nash Response**
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- Frequentist Best Response strategies win lots of money, but are terrible against the wrong opponent
- We'd like a compromise: a strategy that exploits an opponent (or class of opponents), but is also *robust* against arbitrary opponents

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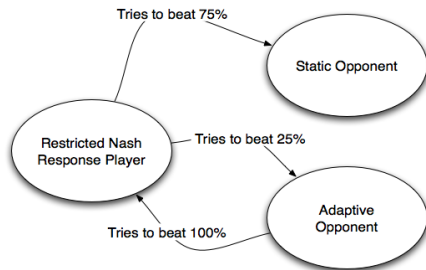
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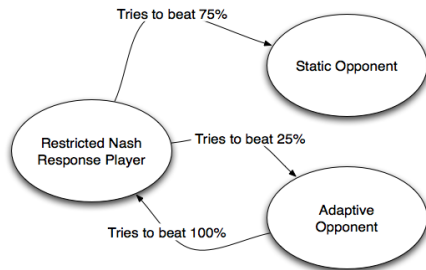
- We suspect our opponent will use some strategy
- What if they only used it, say, 75% of the time?
- The other 25% of the time, they can do anything...
- ...but lets assume they play a best response to whatever we do
- We now have two goals: attack the 75% “weak” strategy, and defend against the 25% “adaptive” strategy

Restricted Nash Response: Basic Idea



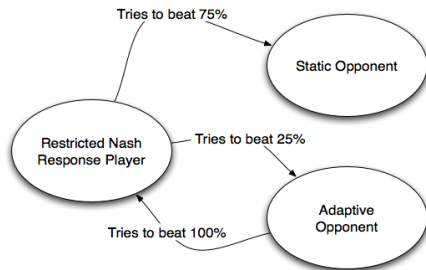
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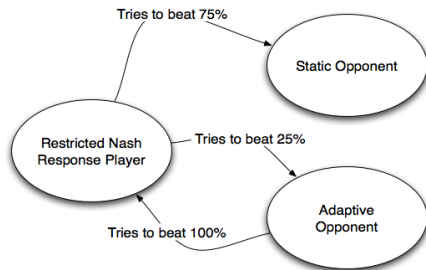
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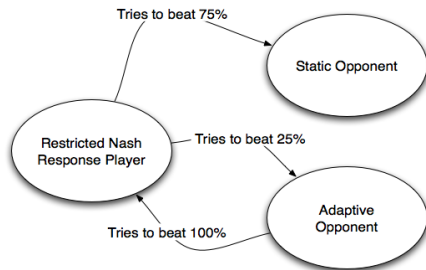
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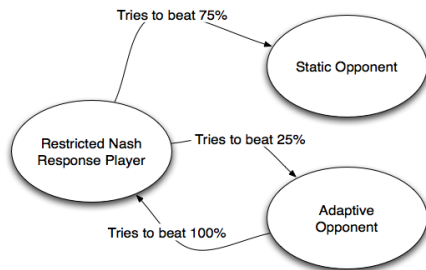
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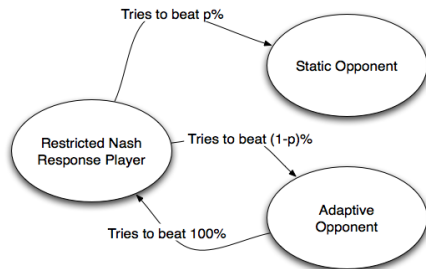
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- The adaptive opponent minimizes regret when playing against us

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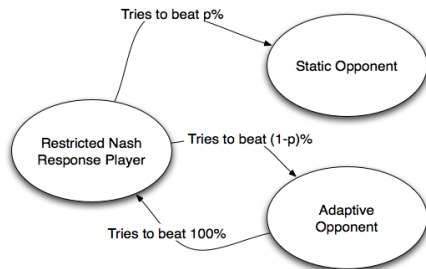
- “Restricted Nash Response”: our opponent is *restricted* to playing the static strategy some of the time.
- We approach a Nash equilibrium in this restricted game.

Restricted Nash Response: Picking the Percentage



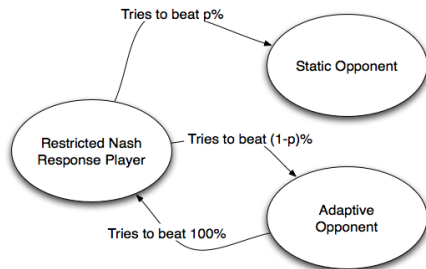
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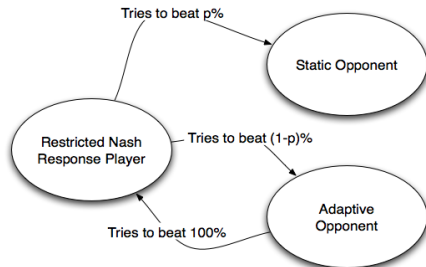
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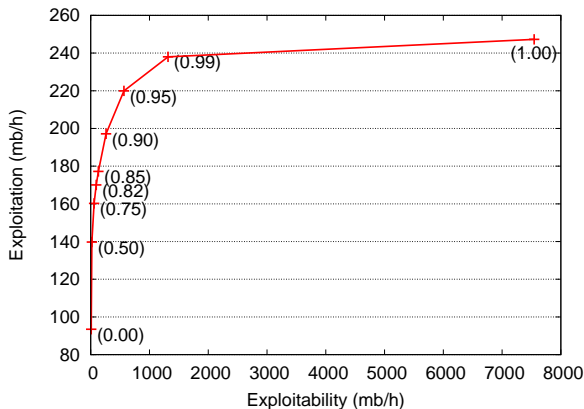
- In the last example, we said the opponent uses the static strategy 75% of the time
- This is actually just a variable, p .
- Interpretations of p :
 - How much you care about exploiting the static strategy
 - How confident you are that the opponent will actually use the static strategy

Restricted Nash Response: Picking the Percentage



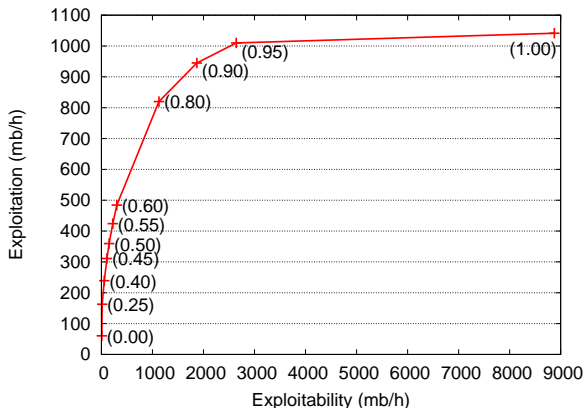
- If p is low, then the resulting counter-strategy is more like a Nash equilibrium
- If p is high, then the resulting counter-strategy is more like a best response

Restricted Nash Response: Picking the Percentage



- X-Axis: How exploitable the counter-strategy is
- Y-Axis: How much we beat the opponent
- Labels: The value of p used to generate the strategy

Restricted Nash Response: Picking the Percentage



Attack80

- Don't use a Nash equilibrium - you can win a lot by giving up a tiny amount!
- Don't use a Best Response - you can save a lot by giving up a tiny amount!

Restricted Nash Response: Results

Frequentist Best Response:

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Restricted Nash Response:

| | Opponents | | | | | | | | Average |
|------------------|-----------|---------|----------|----------|--------------|--------------|--------------|------|---------|
| | PsOpti4 | PsOpti6 | Attack60 | Attack80 | Smallbot1239 | Smallbot1399 | Smallbot2298 | CFR5 | |
| RNR-PsOpti4 | 85 | 112 | 39 | 9 | 63 | 61 | -1 | -23 | 43 |
| RNR-PsOpti6 | 26 | 234 | 72 | 34 | 59 | 59 | 1 | -28 | 57 |
| RNR-Attack60 | -17 | 63 | 582 | -22 | 37 | 39 | -9 | -45 | 78 |
| RNR-Attack80 | -7 | 66 | 22 | 293 | 11 | 12 | 0 | -29 | 46 |
| RNR-Smallbot1239 | 38 | 130 | 68 | 31 | 111 | 106 | 9 | -20 | 59 |
| RNR-Smallbot1399 | 31 | 136 | 66 | 29 | 105 | 112 | 6 | -24 | 58 |
| RNR-Smallbot2298 | 21 | 137 | 72 | 30 | 77 | 76 | 31 | -11 | 54 |
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- Restricted Nash Response makes *robust* counter-strategies
- Exploits one opponent, minimizes weakness against all others
- If you ever have to compute a best response offline, you can do this instead. It’s not so bad if you’re right, and a life saver if you’re wrong.

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 - Used CFR strategies to get a 1st, a 2nd, and a 3rd
- First Man-Machine Poker Championship
 - Played against two poker pros, Phil Laak and Ali Eslami
 - Used CFR and RNR strategies to win one, tie one, and lose two
 - Post-game analysis suggests a different result

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- The next Man-Machine match might have a different outcome!

Questions?



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- 3 Events:
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 - Heads-Up Limit Online Learning
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| | Hyperborean07EQ | IanBot | GS3 | PokeMinn | Quick | Gomel-2 | DumboEQ | DumboEQ-2 | Sequel | Sequel-2 | PokeMinn-2 | UNCC | Gomel | LeRenard | MonashBPP | MilanoEQ | Average |
|-----------------|-----------------|--------|------|----------|-------|---------|---------|-----------|--------|----------|------------|------|-------|----------|-----------|----------|---------|
| Hyperborean07EQ | | 21 | 32 | 136 | 115 | 110 | 193 | 182 | 165 | 166 | 131 | 454 | 115 | 138 | 465 | 428 | 194 |
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| DumboEQ-2 | -182 | -119 | -149 | -76 | -135 | -200 | -133 | | 87 | 82 | 83 | -52 | 271 | 54 | 808 | 762 | 74 |
| Sequel | -165 | -131 | -140 | 33 | -125 | -135 | -67 | -87 | | 19 | 130 | 167 | -17 | 92 | 556 | 566 | 46 |
| Sequel-2 | -166 | -140 | -148 | 22 | -121 | -150 | -64 | -82 | -19 | | 125 | 174 | -4 | 74 | 583 | 526 | 41 |
| PokeMinn-2 | -131 | -142 | -154 | 24 | -134 | -16 | -55 | -83 | -130 | -125 | | 96 | 123 | 60 | 770 | 748 | 57 |
| UNCC | -454 | -472 | -467 | -373 | -298 | -275 | -23 | 52 | -167 | -174 | -96 | | 95 | -281 | 553 | 503 | -125 |
| Gomel | -115 | -88 | -107 | -265 | -149 | -232 | -300 | -271 | 17 | 4 | -123 | -95 | | 96 | 779 | 993 | 10 |
| LeRenard | -138 | -130 | -142 | -127 | -15 | -136 | -13 | -54 | -92 | -74 | -60 | 281 | -96 | | 478 | 354 | 2 |
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| DumboEQ-2 | -182 | -119 | -149 | -76 | -135 | -200 | -133 | | 87 | 82 | 83 | -52 | 271 | 54 | 808 | 762 | 74 |
| Sequel | -165 | -131 | -140 | 33 | -125 | -135 | -67 | -87 | | 19 | 130 | 167 | -17 | 92 | 556 | 566 | 46 |
| Sequel-2 | -166 | -140 | -148 | 22 | -121 | -150 | -64 | -82 | -19 | | 125 | 174 | -4 | 74 | 583 | 526 | 41 |
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| DumboEQ | -193 | -142 | -160 | -80 | -235 | -206 | | 133 | 67 | 64 | 55 | 23 | 300 | 13 | 774 | 672 | 72 |
| DumboEQ-2 | -182 | -119 | -149 | -76 | -135 | -200 | -133 | | 87 | 82 | 83 | -52 | 271 | 54 | 808 | 762 | 74 |
| Sequel | -165 | -131 | -140 | 33 | -125 | -135 | -67 | -87 | | 19 | 130 | 167 | -17 | 92 | 556 | 556 | 46 |
| Sequel-2 | -166 | -140 | -148 | 22 | -121 | -150 | -64 | -82 | -19 | | 125 | 174 | -4 | 74 | 583 | 526 | 41 |
| PokeMinn-2 | -131 | -142 | -154 | 24 | -134 | -16 | -55 | -83 | -130 | -125 | | 96 | 123 | 60 | 770 | 748 | 57 |
| UNCC | -454 | -472 | -467 | -373 | -298 | -275 | -23 | 52 | -167 | -174 | -96 | | 95 | -281 | 553 | 503 | -125 |
| Gomel | -115 | -88 | -107 | -265 | -149 | -232 | -300 | -271 | 17 | 4 | -123 | -95 | | 96 | 779 | 993 | 10 |
| LeRenard | -138 | -130 | -142 | -127 | -15 | -136 | -13 | -54 | -92 | -74 | -60 | 281 | -96 | | 478 | 354 | 2 |
| MonashBPP | -465 | -408 | -412 | -627 | -564 | -802 | -774 | -808 | -556 | -583 | -770 | -553 | -779 | -478 | | 489 | -539 |
| MilanoEQ | -428 | -398 | -445 | -421 | -489 | -859 | -672 | -762 | -556 | -526 | -748 | -503 | -993 | -354 | -489 | | -576 |

AAAI: Heads-Up Limit Online Learning

- Winner determined by total winnings (in dollars)

| | Hyperborean07OL-2 | Hyperborean07OL | GS3 | IanBot | Quick | Gomel-2 | PokeMinn | Sequel | Sequel-2 | LeRenard | DumboOL-2 | Average |
|-------------------|-------------------|-----------------|------|--------|-------|---------|----------|--------|----------|----------|-----------|---------|
| Hyperborean07OL-2 | | -37 | -27 | -37 | 138 | 155 | 172 | 166 | 178 | 170 | 259 | 114 |
| Hyperborean07OL | 37 | | 21 | 27 | 116 | 108 | 141 | 153 | 175 | 132 | 207 | 112 |
| GS3 | 27 | -21 | | 6 | 73 | 112 | 150 | 140 | 148 | 142 | 199 | 98 |
| IanBot | 37 | -27 | -6 | | 99 | 85 | 130 | 131 | 140 | 130 | 157 | 87 |
| Quick | -138 | -116 | -73 | -99 | | 19 | -40 | 125 | 121 | 15 | 129 | -6 |
| Gomel-2 | -155 | -108 | -112 | -85 | -19 | | -144 | 135 | 150 | 136 | 123 | -8 |
| PokeMinn | -172 | -141 | -150 | -130 | 40 | 144 | | -33 | -22 | 127 | -15 | -35 |
| Sequel | -166 | -153 | -140 | -131 | -125 | -135 | 33 | | 19 | 92 | -1 | -71 |
| Sequel-2 | -178 | -175 | -148 | -140 | -121 | -150 | 22 | -19 | | 74 | 17 | -82 |
| LeRenard | -170 | -132 | -142 | -130 | -15 | -136 | -127 | -92 | -74 | | 21 | -100 |
| DumboOL-2 | -259 | -207 | -199 | -157 | -129 | -123 | 15 | 1 | -17 | -21 | | -110 |

AAAI: Heads-Up Limit Online Learning

- Winner determined by total winnings (in dollars)
- Took second place with a CFR bot. We just barely lost to...

| | Hyperborean07OL-2 | Hyperborean07OL | GS3 | IanBot | Quick | Gomel-2 | PokeMinn | Sequel | Sequel-2 | LeRenard | DumboOL-2 | Average |
|-------------------|-------------------|-----------------|------|--------|-------|---------|----------|--------|----------|----------|-----------|---------|
| Hyperborean07OL-2 | | -37 | -27 | -37 | 138 | 155 | 172 | 166 | 178 | 170 | 259 | 114 |
| Hyperborean07OL | 37 | | 21 | 27 | 116 | 108 | 141 | 153 | 175 | 132 | 207 | 112 |
| GS3 | 27 | -21 | | 6 | 73 | 112 | 150 | 140 | 148 | 142 | 199 | 98 |
| IanBot | 37 | -27 | -6 | | 99 | 85 | 130 | 131 | 140 | 130 | 157 | 87 |
| Quick | -138 | -116 | -73 | -99 | | 19 | -40 | 125 | 121 | 15 | 129 | -6 |
| Gomel-2 | -155 | -108 | -112 | -85 | -19 | | -144 | 135 | 150 | 136 | 123 | -8 |
| PokeMinn | -172 | -141 | -150 | -130 | 40 | 144 | | -33 | -22 | 127 | -15 | -35 |
| Sequel | -166 | -153 | -140 | -131 | -125 | -135 | 33 | | 19 | 92 | -1 | -71 |
| Sequel-2 | -178 | -175 | -148 | -140 | -121 | -150 | 22 | -19 | | 74 | 17 | -82 |
| LeRenard | -170 | -132 | -142 | -130 | -15 | -136 | -127 | -92 | -74 | | 21 | -100 |
| DumboOL-2 | -259 | -207 | -199 | -157 | -129 | -123 | 15 | 1 | -17 | -21 | | -110 |

AAAI: Heads-Up Limit Online Learning

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- Took second place with a CFR bot. We just barely lost to...
- ...the other U of A bot

| | Hyperborean07OL-2 | Hyperborean07OL | GS3 | IanBot | Quick | Gomel-2 | PokeMinn | Sequel | Sequel-2 | LeRenard | DumboOL-2 | Average |
|-------------------|-------------------|-----------------|------|--------|-------|---------|----------|--------|----------|----------|-----------|---------|
| Hyperborean07OL-2 | | -37 | -27 | -37 | 138 | 155 | 172 | 166 | 178 | 170 | 259 | 114 |
| Hyperborean07OL | 37 | | 21 | 27 | 116 | 108 | 141 | 153 | 175 | 132 | 207 | 112 |
| GS3 | 27 | -21 | | 6 | 73 | 112 | 150 | 140 | 148 | 142 | 199 | 98 |
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| Sequel | -166 | -153 | -140 | -131 | -125 | -135 | 33 | | 19 | 92 | -1 | -71 |
| Sequel-2 | -178 | -175 | -148 | -140 | -121 | -150 | 22 | -19 | | 74 | 17 | -82 |
| LeRenard | -170 | -132 | -142 | -130 | -15 | -136 | -127 | -92 | -74 | | 21 | -100 |
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- Winner determined by total winnings (in dollars)
- Took second place with a CFR bot. We just barely lost to...
- ...the other U of A bot
- (Darse Billings and Morgan Kan)

| | Hyperborean07OL-2 | Hyperborean07OL | GS3 | IanBot | Quick | Gomel-2 | PokeMinn | Sequel | Sequel-2 | LeRenard | DumboOL-2 | Average |
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| Gomel-2 | -155 | -108 | -112 | -85 | -19 | | -144 | 135 | 150 | 136 | 123 | -8 |
| PokeMinn | -172 | -141 | -150 | -130 | 40 | 144 | | -33 | -22 | 127 | -15 | -35 |
| Sequel | -166 | -153 | -140 | -131 | -125 | -135 | 33 | | 19 | 92 | -1 | -71 |
| Sequel-2 | -178 | -175 | -148 | -140 | -121 | -150 | 22 | -19 | | 74 | 17 | -82 |
| LeRenard | -170 | -132 | -142 | -130 | -15 | -136 | -127 | -92 | -74 | | 21 | -100 |
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AAAI: Heads-Up No-Limit

- No-Limit is what you see on TV - bets can be any size

| | BluffBot20 | GS3 | Hyperborean07 | SlideRule | Gomel | Gomel-2 | Milano | Manitoba | PokeMinn | Manitoba-2 | Average |
|---------------|------------|-------|---------------|-----------|--------|---------|--------|----------|----------|------------|---------|
| BluffBot20 | | 267 | 380 | 576 | 2093 | 2885 | 3437 | 475 | 1848 | 2471 | 1603 |
| GS3 | -267 | | 113 | 503 | 3161 | 124 | 1875 | 4204 | -42055 | 5016 | -3036 |
| Hyperborean07 | -380 | -113 | | -48 | 6657 | 5455 | 6795 | 8697 | 12051 | 22116 | 6803 |
| SlideRule | -576 | -503 | 48 | | 11596 | 9730 | 10337 | 10387 | 15637 | 10791 | 7494 |
| Gomel | -2093 | -3161 | -6657 | -11596 | | 3184 | 8372 | 11450 | 62389 | 52325 | 12690 |
| Gomel-2 | -2885 | -124 | -5455 | -9730 | -3184 | | 15078 | 11907 | 58985 | 40256 | 11650 |
| Milano | -3437 | -1875 | -6795 | -10337 | -8372 | -15078 | | 5741 | 12719 | 27040 | -44 |
| Manitoba | -475 | -4204 | -8697 | -10387 | -11450 | -11907 | -5741 | | 18817 | 50677 | 1848 |
| PokeMinn | -1848 | 42055 | -14051 | -15637 | -62389 | -58985 | -12719 | -18817 | | 34299 | -12010 |
| Manitoba-2 | -2471 | -5016 | -22116 | -10791 | -52325 | -40256 | -27040 | -50677 | -34299 | | -27221 |

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- This was our first time making a No-Limit bot

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|---------------|------------|-------|---------------|-----------|--------|---------|--------|----------|----------|------------|---------|
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| Gomel | -2093 | -3161 | -6657 | -11596 | | 3184 | 8372 | 11450 | 62389 | 52325 | 12690 |
| Gomel-2 | -2885 | -124 | -5455 | -9730 | -3184 | | 15078 | 11907 | 58985 | 40256 | 11650 |
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| Manitoba | -475 | -4204 | -8697 | -10387 | -11450 | -11907 | -5741 | | 18817 | 50677 | 1848 |
| PokeMinn | -1848 | 42055 | -14051 | -15637 | -62389 | -58985 | -12719 | -18817 | | 34299 | -12010 |
| Manitoba-2 | -2471 | -5016 | -22116 | -10791 | -52325 | -40256 | -27040 | -50677 | -34299 | | -27221 |

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- Took third place, using a CFR bot with abstracted betting

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|---------------|------------|-------|---------------|-----------|--------|---------|--------|----------|----------|------------|---------|
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| Gomel | -2093 | -3161 | -6657 | -11596 | | 3184 | 8372 | 11450 | 62389 | 52325 | 12690 |
| Gomel-2 | -2885 | -124 | -5455 | -9730 | -3184 | | 15078 | 11907 | 58985 | 40256 | 11650 |
| Milano | -3437 | -1875 | -6795 | -10337 | -8372 | -15078 | | 5741 | 12719 | 27040 | -44 |
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AAAI: Heads-Up No-Limit

- No-Limit is what you see on TV - bets can be any size
- This was our first time making a No-Limit bot
- Took third place, using a CFR bot with abstracted betting
- We hope to do better next year! Lots of exciting work to be done here.

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|---------------|------------|-------|---------------|-----------|--------|---------|--------|----------|----------|------------|---------|
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First Man-Machine Poker Championship



- Beating human experts is a big milestone

First Man-Machine Poker Championship



- Beating human experts is a big milestone
- Tough to get statistical significance against humans

First Man-Machine Poker Championship



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- So we played two at once with the same cards

First Man-Machine Poker Championship



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- Tough to get statistical significance against humans
- So we played two at once with the same cards
- Four matches of 500 hands each

First Man-Machine Poker Championship



- Beating human experts is a big milestone
- Tough to get statistical significance against humans
- So we played two at once with the same cards
- Four matches of 500 hands each
- Have to be ahead by 25 small bets to win a match



- Background: Mechanical Engineer
- Started gambling in competitive backgammon
- Competes in the world's biggest poker tournaments



- Background: Computer consultant
- Started out by playing...
- Plays in \$1000-\$2000 Limit games



- Background: Computer consultant
- Started out by playing... Magic: The Gathering
- Plays in \$1000-\$2000 Limit games



- Background: Computer consultant
- Started out by playing... Magic: The Gathering
- Plays in \$1000-\$2000 Limit games
- (This is a lot of money!)

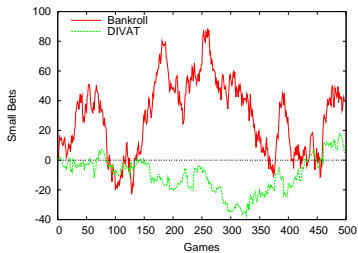


- We had 10 different bots to use:
 - Several Counterfactual Regret Minimization approximate Nash equilibria
 - Flavours of Restricted Nash Response counter-strategies

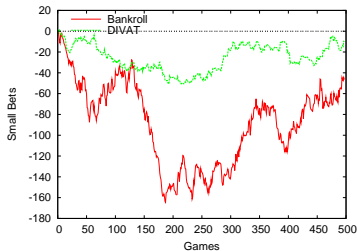


- We had 10 different bots to use:
 - Several Counterfactual Regret Minimization approximate Nash equilibria
 - Flavours of Restricted Nash Response counter-strategies
- We wanted a baseline to compare future bots against
- Bot used: Mr. Pink, our finest abstraction CFR approximate Nash equilibrium

Day 1, Session 1

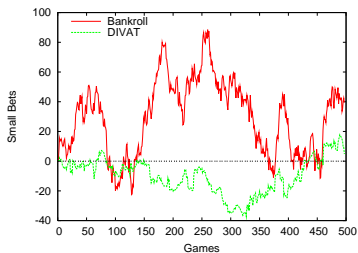


On Stage: Ali Eslami

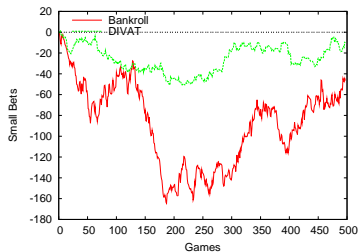


Hotel: Phil Laak

Day 1, Session 1



On Stage: Ali Eslami



Hotel: Phil Laak

- Ali: \$395
- Phil: -\$465
- Polaris ends ahead by \$70
- Result: Tie

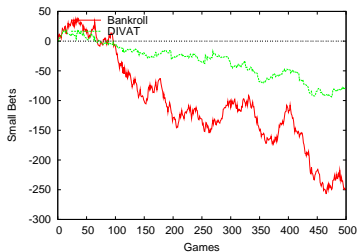


- Score so far: 1 Tie
- The careful choice (Mr. Pink) did OK, so lets try something crazy!

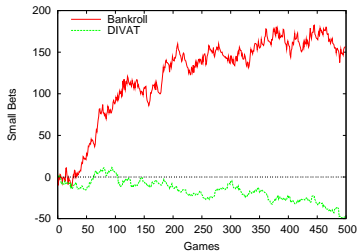


- Score so far: 1 Tie
- The careful choice (Mr. Pink) did OK, so lets try something crazy!
- Bot used: Mr. Orange / Crazy 8s
- It's a CFR approximate Nash equilibrium in a broken game that encourages aggression

Day 1, Session 2

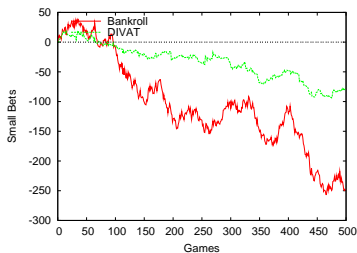


Hotel: Ali Eslami

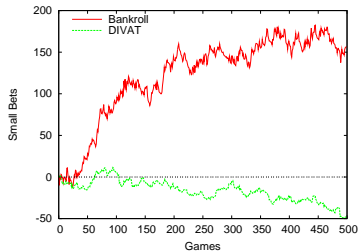


On Stage: Phil Laak

Day 1, Session 2



Hotel: Ali Eslami



On Stage: Phil Laak

- Ali: -\$2495
- Phil: \$1570
- Polaris ends ahead by \$925
- Result: Win



- Score so far: 1 Win, 1 Tie
- Which of our 10 bots to use this time?

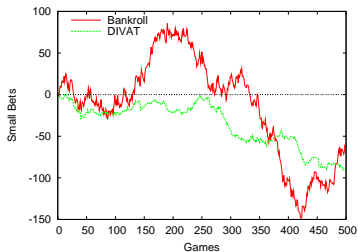


- Score so far: 1 Win, 1 Tie
- Which of our 10 bots to use this time?
- We pulled an all nighter and ran importance sampling on the last 1000 hands
- Predicted the best 3 bots to use against each player

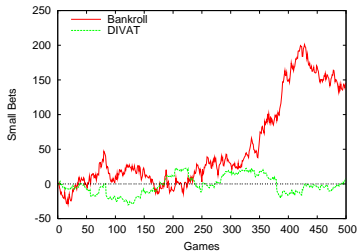


- Score so far: 1 Win, 1 Tie
- Which of our 10 bots to use this time?
- We pulled an all nighter and ran importance sampling on the last 1000 hands
- Predicted the best 3 bots to use against each player
- Used a coach that chose between these 3 during the match

Day 2, Session 1

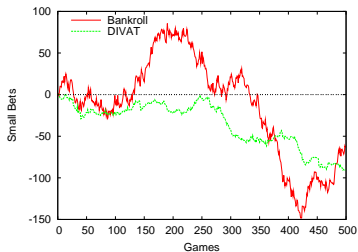


Hotel: Ali Eslami

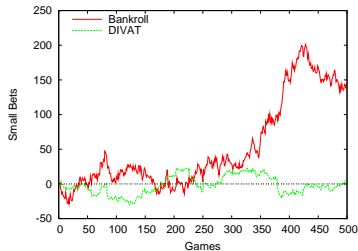


On Stage: Phil Laak

Day 2, Session 1



Hotel: Ali Eslami



On Stage: Phil Laak

- Ali: -\$635
- Phil: \$1455
- Polaris ends behind by \$820
- Result: Loss

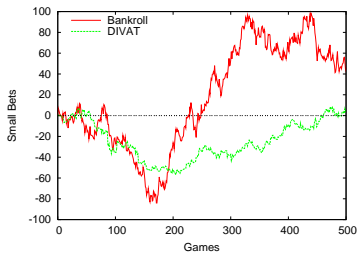


- Score so far: 1 Win, 1 Tie, 1 Loss
- Decided to play it safe and go for a tie

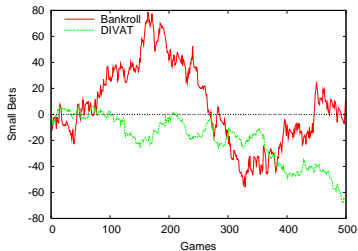


- Score so far: 1 Win, 1 Tie, 1 Loss
- Decided to play it safe and go for a tie
- Bot used: Mr. Pink, the approximate Nash equilibrium from the first match

Day 2, Session 2

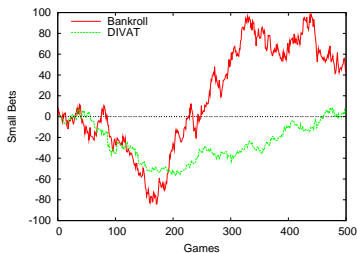


Onstage: Ali Eslami

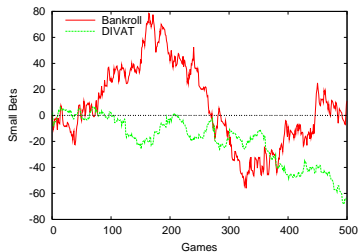


Hotel: Phil Laak

Day 2, Session 2



Onstage: Ali Eslami



Hotel: Phil Laak

- Ali: \$460
- Phil: \$110
- Polaris ends behind by \$570
- Result: Loss

Man-Machine Match Conclusions

- Very close game — we lost by 0.01 small bets/game, less than the tie margin

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- Ali: “This was not a win for us...I played the best heads-up poker I’ve ever played...we just barely won”

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- There’s lots of exciting work to do here, too!